19.1 Pythagoras’ theorem

Pythagoras was a famous mathematician in Ancient Greece. The theorem which is named after him is an important result about right-angled triangles.

Here is a right-angled triangle $ABC$.

The angle at $C$ is the right angle. The side, $AB$, opposite the right angle is called the hypotenuse. It is the longest side in the triangle.

The right-angled triangle in the diagram on the right has sides of length 3 cm, 4 cm and 5 cm. Squares have been drawn on each side of the triangle and each square has been divided up into squares of side 1 cm.

The area of the square on the side of length 3 cm is 9 cm$^2$. The area of the square on the side of length 4 cm is 16 cm$^2$. The area of the square on the side of length 5 cm (the hypotenuse) is 25 cm$^2$.

Notice that $25 \text{ cm}^2 = 9 \text{ cm}^2 + 16 \text{ cm}^2$

that is, $5^2 = 3^2 + 4^2$

In other words $5^2$ (the area of the square on the hypotenuse) is equal to the sum of $3^2$ and $4^2$ (the areas of the squares on the other two sides added together).

This is an example of Pythagoras’ theorem. It is only true for right-angled triangles.

Pythagoras’ theorem states:

In a right-angled triangle, the area of the square on the hypotenuse is equal to the sum of areas of the squares on the other two sides.

Area of square $R = \text{area of square } P + \text{area of square } Q$

Pythagoras’ theorem can be used to find the length of the third side of a right-angled triangle when the lengths of the other two sides are known. For this, the theorem is usually stated in terms of the lengths of the sides of the triangle.

That is

$c^2 = a^2 + b^2$

Pythagoras’ theorem can also be written

$DE^2 = EF^2 + DF^2$

($DE^2$ means that the length of the side $DE$ is squared.)
19.2 Finding lengths

Pythagoras' theorem can be used to work out the length of the hypotenuse of a right-angled triangle when the lengths of the two shorter sides are given.

Example 1

Work out the length of the hypotenuse in this triangle.

\[ c^2 = a^2 + b^2 \]
\[ c^2 = 8^2 + 15^2 \]
\[ c^2 = 64 + 225 \]
\[ c^2 = 289 \]
\[ c = \sqrt{289} = 17 \]

Length of hypotenuse = 17 cm

Solution 1
State Pythagoras' theorem.
Substitute the given lengths.
Work out \( a^2 \) and \( b^2 \) and add the results.
Find \( \sqrt{289} \)
The answer is sensible because the hypotenuse is longer than the other two sides.

It is important to be able to apply Pythagoras' theorem when the triangle is in a different position.

Example 2

In triangle \( XYZ \), angle \( X = 90^\circ \), \( XY = 8.6 \) cm and \( XZ = 13.9 \) cm.
Work out the length of \( YZ \).
Give your answer correct to 3 significant figures.

\[ YZ^2 = XY^2 + XZ^2 \]
\[ YZ^2 = 8.6^2 + 13.9^2 \]
\[ YZ^2 = 73.96 + 193.21 \]
\[ YZ^2 = 267.17 \]
\[ YZ = \sqrt{267.17} = 16.34... \]
\[ YZ = 16.3 \text{ cm (to 3 s.f.)} \]

Solution 2
State Pythagoras' theorem.
Substitute the given lengths.
Work out \( 8.6^2 \) and \( 13.9^2 \) and add the results.
Find \( \sqrt{267.17} \) Write down at least 4 figures.
Give the final answer correct to 3 significant figures.
Pythagoras’ theorem can also be used to work out the length of one of the shorter sides in a right-angled triangle when the lengths of the other two sides are known.

**Example 3**

In triangle $ABC$, angle $A = 90^\circ$, $BC = 17.4\,\text{cm}$ and $AC = 5.8\,\text{cm}$. Work out the length of $AB$. Give your answer correct to 3 significant figures.

**Solution 3**

Angle $A$ is the right angle so the hypotenuse is $BC$.

$$BC^2 = AC^2 + AB^2$$

Substitute the given lengths.

$$17.4^2 = 5.8^2 + AB^2$$

Work out $17.4^2$ and $5.8^2$.

$$302.76 = 33.64 + AB^2$$

Subtract $33.64$ from both sides.

$$269.12 = AB^2$$

Find $\sqrt{269.12}$ Write down at least 4 figures.

$$AB = 16.40...$$

Give the final answer correct to 3 significant figures.

$$AB = 16.4\,\text{cm}$$ (to 3 s.f.)

**Exercise 19A**

1. Work out the length of the sides marked with letters in these triangles.

   a) [Diagram](#)

   b) [Diagram](#)

   c) [Diagram](#)

2. Work out the length of the sides marked with letters in these triangles.

   a) [Diagram](#)

   b) [Diagram](#)

   c) [Diagram](#)

3. Work out the length of the sides marked with letters in these triangles. Give each answer correct to 3 significant figures.

   a) [Diagram](#)

   b) [Diagram](#)

   c) [Diagram](#)

   d) [Diagram](#)
4 Work out the length of the sides marked with letters in these triangles. Give each answer correct to 3 significant figures.

\[ \text{a) } \triangle ABC \text{ with } AB = 4.8 \text{ cm and } AC = 10.7 \text{ cm} \]

\[ \text{b) } \triangle ABC \text{ with } AB = 8.1 \text{ cm and } AC = 11.3 \text{ cm} \]

\[ \text{c) } \triangle ABC \text{ with } AB = 2.1 \text{ cm and } AC = 12.4 \text{ cm} \]

\[ \text{d) } \triangle ABC \text{ with } AB = 5.9 \text{ cm and } AC = 16.3 \text{ cm} \]

\[ \text{e) } \triangle ABC \text{ with } AB = 3.4 \text{ cm and } AC = 12.1 \text{ cm} \]

\[ \text{f) } \triangle ABC \text{ with } AB = 1.8 \text{ cm and } AC = 12.4 \text{ cm} \]

5 a In triangle \( ABC \)
angle \( A = 90^\circ \), \( AB = 3.4 \text{ cm} \) and \( AC = 12.1 \text{ cm} \).
Work out the length of \( BC \).
Give your answer correct to 3 significant figures.

b In triangle \( ABC \)
angle \( A = 90^\circ \), \( AB = 5.9 \text{ cm} \) and \( BC = 16.3 \text{ cm} \).
Work out the length of \( AC \).
Give your answer correct to 3 significant figures.

c In triangle \( PQR \)
angle \( R = 90^\circ \), \( PR = 5.9 \text{ cm} \) and \( QR = 13.1 \text{ cm} \).
Work out the length of \( PQ \).
Give your answer correct to 3 significant figures.

d In triangle \( PQR \)
angle \( R = 90^\circ \), \( PQ = 11.2 \text{ cm} \) and \( QR = 9.6 \text{ cm} \).
Work out the length of \( RP \).
Give your answer correct to 3 significant figures.

e In triangle \( XYZ \)
angle \( X = 90^\circ \), \( XY = 12.6 \text{ cm} \) and \( XZ = 16.5 \text{ cm} \).
Work out the length of \( YZ \).
Give your answer correct to 3 significant figures.

f In triangle \( DEF \)
angle \( E = 90^\circ \), \( DF = 10.1 \text{ cm} \) and \( EF = 7.8 \text{ cm} \).

i Draw a sketch of the right-angled triangle \( DEF \) and label sides \( DF \) and \( EF \) with their lengths.

ii Work out the length of \( DE \).
Give your answer correct to 3 significant figures.
19.3 Applying Pythagoras' theorem

Pythagoras' theorem can be used to solve problems.

**Example 4**

A boat travels due north for 5.7 km. The boat then turns and travels due east for 7.2 km. Work out the distance between the boat's finishing point and its starting point. Give your answer correct to 3 significant figures.

**Solution 4**

\[ d^2 = 5.7^2 + 7.2^2 \]
\[ d^2 = 32.49 + 51.84 \]
\[ d^2 = 84.33 \]
\[ d = \sqrt{84.33} = 9.183... \]
Distance = 9.18 km (to 3 s.f.)

Isosceles triangles can be split into two right-angled triangles and Pythagoras' theorem can then be used.

**Example 5**

The diagram shows an isosceles triangle \( ABC \). The midpoint of \( BC \) is the point \( M \).

In the triangle, \( AB = AC = 8 \text{ cm} \) and \( BC = 6 \text{ cm} \).

a) Work out the height, \( AM \), of the triangle.
   Give your answer correct to 3 significant figures.

b) Work out the area of triangle \( ABC \).
   Give your answer correct to 3 significant figures.

**Solution 5**

Pythagoras' theorem cannot be used in triangle \( ABC \) as this triangle is not right-angled.

\[ AB^2 = AM^2 + BM^2 \]
\[ 8^2 = h^2 + 3^2 \]
\[ 64 = h^2 + 9 \]
\[ 64 - 9 = h^2 \]
\[ 55 = h^2 \]
\[ h = \sqrt{55} = 7.416... \]
\[ h = 7.42 \]

Height of triangle = 7.42 cm (to 3 s.f.)
Exercise 19B

1. The diagram shows a ladder leaning against a vertical wall. The foot of the ladder is on horizontal ground. The length of the ladder is 5 m. The foot of the ladder is 3.6 m from the wall. Work out how far up the wall the ladder reaches. Give your answer correct to 3 significant figures.

2. The diagram shows a rectangle of length 9 cm and width 6 cm. Work out the length of a diagonal of the rectangle. Give your answer correct to 3 significant figures.

3. Aiton (A), Beeville (B) and Ceaborough (C) are three towns as shown in this diagram. Beeville is 10 km due south of Aiton and 21 km due east of Ceaborough. Work out the distance between Aiton and Ceaborough. Give your answer correct to the nearest km.

4. Work out the area of the triangle. Give your answer correct to 3 significant figures.

5. Work out the perimeter of the triangle. Give your answer correct to 3 significant figures.

6. The diagram represents the end view of a tent, triangle ABC, two guy-ropes, AP and AQ, and a vertical tent pole, AN. The tent is on horizontal ground so that PBNCQ is a straight horizontal line. Triangles ABC and APQ are both isosceles triangles. BN = NC = 2 m, AN = 2.5 m and AP = AQ = 5 m

   a. Work out the length of the side AC of the tent. Give your answer correct to 3 significant figures.

   b. Work out the length of
      i. NQ  
      ii. CQ.  
      Give your answers correct to 3 significant figures.

There is a tent peg at P and a tent peg at Q.

   c. Work out the distance between the two tent pegs at P and Q. Give your answer correct to 3 significant figures.
7 The diagram shows two right-angled triangles.
   a Work out the length of the side marked \( x \).
   b Hence work out the length of the side marked \( y \).

8 The diagram shows two right-angled triangles.
   Work out the length of the side marked \( a \).
   Give your answer correct to 3 significant figures.

9 The lengths of the sides of a triangle are 8 cm, 15 cm and 17 cm.
   a Find the value of \( i \) \( 8^2 + 15^2 \)
      \( ii \) \( 17^2 \)
   b What do you notice about the two answers in a?
   c What information does this give about the triangle?

10 Here are the lengths of sides of six triangles.

<table>
<thead>
<tr>
<th>Triangle 1</th>
<th>5 cm, 12 cm and 13 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle 2</td>
<td>9 cm, 40 cm and 41 cm</td>
</tr>
<tr>
<td>Triangle 3</td>
<td>10 cm, 17 cm and 18 cm</td>
</tr>
<tr>
<td>Triangle 4</td>
<td>20 cm, 21 cm and 29 cm</td>
</tr>
<tr>
<td>Triangle 5</td>
<td>8 cm, 17 cm and 20 cm</td>
</tr>
<tr>
<td>Triangle 6</td>
<td>33 cm, 56 cm and 65 cm</td>
</tr>
</tbody>
</table>

Find by calculation which of these triangles are right-angled triangles.

19.4 Coordinates, line segments and Pythagoras’ theorem

The diagram shows the straight line joining the points \( A(2, 1) \) and \( B(8, 5) \). A line joining two points is called a line segment. So in the diagram, \( AB \) is the line segment joining the points \( A \) and \( B \).

Midpoint of a line segment

The midpoint \( M \) of the line segment \( AB \) has coordinates \( (5, 3) \).

Notice that the \( x \)-coordinate of \( A \) is 2, the \( x \)-coordinate of \( B \) is 8 and \( \frac{2 + 8}{2} = 5 \), the \( x \)-coordinate of \( M \).

Similarly the \( y \)-coordinate of \( A \) is 1, the \( y \)-coordinate of \( B \) is 5 and \( \frac{1 + 5}{2} = 3 \), the \( y \)-coordinate of \( M \).

In general the \( x \)-coordinate of the midpoint of a line segment is the mean of the \( x \)-coordinates of its endpoints and the \( y \)-coordinate of its midpoint is the mean of the \( y \)-coordinates of its endpoints.

That is

the midpoint of the line joining \( (a, b) \) and \( (p, q) \) is the point \( \left( \frac{a + p}{2}, \frac{b + q}{2} \right) \)
Example 6

Find the midpoint of the line joining
a \((3, 5)\) and \((13, 7)\)  

**Solution 6**  
\[a \quad 3 + 13 = 16\]  
\[16 \div 2 = 8\]  
\[5 + 7 = 12\]  
\[12 \div 2 = 6\]  
Midpoint is \((8, 6)\)

\[b \quad -5 + 9 = 4\]  
\[4 \div 2 = 2\]  
\[8 + (-13) = -5\]  
\[-5 \div 2 = -2.5\]  
Midpoint is \((2, -2.5)\)

Length of a line segment

The diagram shows the points \(A(1, 1)\) and \(B(9, 5)\).  
The right-angled triangle \(ABC\) has been drawn so that \(AC = 8\) and \(BC = 4\)

Pythagoras' theorem can be used to find the length of \(AB\).

\[AB^2 = 8^2 + 4^2\]  
\[AB^2 = 64 + 16 = 80\]  
\[AB = \sqrt{80} = 8.94\] (to 3 s.f.)

Example 7

Find the length of the line joining
a \(A(3, 2)\) and \(B(15, 7)\)

**Solution 7**  
\[y\]

\[x\]

\[AB^2 = 12^2 + 5^2\]  
\[AB^2 = 144 + 25 = 169\]  
\[AB = \sqrt{169} = 13\]
Exercise 19C

1. Work out the coordinates of the midpoint of the line joining
   a. (3, 1) and (11, 7)
   b. (2, 5) and (12, 29)
   c. (−6, 9) and (8, 13)
   d. (−4, −6) and (6, 12)
   e. (9, −15) and (−11, 6)
   f. (0, −5) and (9, −11)

2. Work out the length of the line joining each of the pair of points in question 1

3. The point A has coordinates (5, 2), the point B has coordinates (8, 6) and the point C has coordinates (1, 5).
   a. Work out the length of
      i. AB
      ii. BC
      iii. AC.
   b. What does your answer to part a tell you about triangle ABC?

4. The point P has coordinates (5, 3), the point Q has coordinates (6, 6) and the point R has coordinates (6, 10).
   a. Work out the length of each side of triangle PQR.
   b. Use your answers to part a to show that triangle PQR is a right-angled triangle.
   c. Work out the area of triangle PQR.

5. The points A(2, 6) and B(18, 36) are the ends of a diameter of a circle.
   a. Find the coordinates of the centre of the circle.
   b. Work out the
      i. diameter of the circle
      ii. radius of the circle.

6. A circle has centre O(4, 2). The point A(9, 14) lies on the circle.
   a. Work out the radius of the circle.
   b. Determine by calculation which of the following points also lie on the circle.
      i. B(16, 7)
      ii. C(−1, −10)
      iii. D(7, 16)

7. The point A has coordinates (−3, −8) and the point B has coordinates (8, 9).
   a. Find the coordinates of the midpoint of the line segment AB.
   b. Work out the length of the line segment AB. Give your answer correct to 3 significant figures.

8. The points A (5, 1), B (29, 8), C (9, 23) and D (−15, 16) are the vertices of quadrilateral ABCD.
   a. Work out the length of
      i. AB
      ii. BC
      iii. CD
      iv. DA.
   b. Explain what your answers to a tell you about the quadrilateral ABCD.

9. The point A has coordinates (a, b) and the point B has coordinates (p, q).
   Show that the length of the line segment AB is \( \sqrt{(p - a)^2 + (q - b)^2} \)
19.5 Trigonometry – introduction

Trigonometry means ‘triangle measure’. It is used to work out lengths and angles in triangles and in shapes that can be divided up into triangles. Trigonometry is important in bridge building and tunnel building where it is important to know accurate distances and accurate angle sizes. It is also used in many other areas of surveying, engineering and architecture.

**Trigonometric ratios**

Here are two right-angled triangles.

The triangle with hypotenuse 2 is an enlargement with scale factor 2 of the triangle with hypotenuse 1, that is, its sides are twice as long.

\[ p = 2s \quad \text{and} \quad q = 2c \]

So if \( s \) and \( c \) are known for the right-angled triangle with hypotenuse 1, \( p \) and \( q \) can be calculated.

If the length of its hypotenuse is known, the lengths of the sides of any right-angled triangle which is an enlargement of these triangles can be calculated.

The values of \( s \) and \( c \) are known accurately and can be found on any standard scientific calculator.

The length \( s \) is called the sine of 70° written \( \sin 70° \). Not all calculators are the same but the following key sequence to find \( \sin 70° \) applies to many calculators.

Make sure that the angle mode of your calculator is degrees, usually shown by \( \text{D} \) on the calculator screen.

```
Press sin Key in 70 Press =
```

The number 0.93969262 should appear on your calculator display.

So correct to 4 decimal places \( \sin 70° = 0.9397 \)

The length \( c \) is called the cosine of 70° and is written \( \cos 70° \). As above but using the button \( \cos \) correct to 4 decimal places \( \cos 70° = 0.3420 \)

Using the triangles opposite and writing \( s \) as \( \sin 70° \) and \( c \) as \( \cos 70° \)

\[ p = 2\sin 70° \quad \text{and} \quad q = 2\cos 70° \]

So for any right-angled triangle

\[ p = r \sin x° \quad \text{and} \quad q = r \cos x° \]

or \( \sin x° = \frac{p}{r} \) and \( \cos x° = \frac{q}{r} \)
The hypotenuse \((\text{hyp})\) of a right-angled triangle is the side opposite the right angle and is the longest side of the triangle. The sides of the triangle are named according to their position relative to the angle given or the angle to be found. If this angle is \(x^\circ\) then the side opposite this angle is called the opposite side \((\text{opp})\). The side next to this angle is called the adjacent side \((\text{adj})\).

The results above become
\[
\text{opp} = \text{hyp} \times \sin x^\circ \quad \text{and} \quad \text{adj} = \text{hyp} \times \cos x^\circ
\]
or
\[
\sin x^\circ = \frac{\text{opp}}{\text{hyp}} \quad \text{and} \quad \cos x^\circ = \frac{\text{adj}}{\text{hyp}}
\]

\(\sin x^\circ\) is used when opposite and hypotenuse are involved.
\(\cos x^\circ\) is used when adjacent and hypotenuse are involved.

When opposite and adjacent are involved, a third result called \(\tan x^\circ\), is needed where
\[
\tan x^\circ = \frac{\text{opp}}{\text{adj}} \quad \text{or} \quad \text{opp} = \text{adj} \times \tan x^\circ
\]
\(\tan x^\circ\) means the tangent of \(x^\circ\).

**SOHCAHTOA** might help you to remember these results.

\[
\begin{array}{c}
\sin \quad \text{Opp} \\
\text{Hyp} \quad \cos \quad \text{Adj} \\
\text{Tan} \quad \text{Opp} \\
\text{Adj}
\end{array}
\]

**19.6 Finding lengths using trigonometry**

**Example 8**

Work out the length of each of the marked sides. Give each answer correct to 3 significant figures.

**Solution 8**

\(a\)

\[
a = 13 \times \cos 50^\circ = 13 \cos 50^\circ
\]

\(a = 13 \times 0.6427...\)

\(a = 8.356...\)

\(a = 8.36\) cm

13 cm is the hypotenuse
\(a\) is adjacent to the 50\(^\circ\) angle.
adj and hyp are involved so use cos

\[
\cos = \frac{\text{adj}}{\text{hyp}} \quad \text{or} \quad \text{adj} = \text{hyp} \times \cos
\]

Give your answer correct to 3 significant figures.
In triangle $ABC$, angle $CAB = 90^\circ$, angle $ABC = 37^\circ$ and $AB = 8.4$ cm. Calculate the length of $BC$. Give your answer correct to 3 significant figures.

**Solution 9**

In triangle $ABC$, $BC$ is the hypotenuse and $8.4$ cm is adjacent to the $37^\circ$ angle so use $\cos = \frac{\text{adj}}{\text{hyp}}$ or $\text{adj} = \text{hyp} \times \cos$

Substitute the known values in $\text{adj} = \text{hyp} \times \cos$

Make $BC$ the subject.

As $BC$ is the hypotenuse its length must be greater than $8.4$ cm so this is a sensible answer.
Exercise 19D

1 Use a calculator to find the value of each of the following. Give each answer correct to 4 decimal places, where necessary.

- a \( \sin 20^\circ \)
- b \( \sin 72^\circ \)
- c \( \cos 60^\circ \)
- d \( \tan 86^\circ \)
- e \( \tan 45^\circ \)
- f \( \cos 18.9^\circ \)
- g \( \tan 4^\circ \)
- h \( \sin 14.7^\circ \)
- i \( \cos 75.3^\circ \)

2 Work out the lengths of the sides marked with letters. Give each answer correct to 3 significant figures.

3 Triangle \( PQR \) is right-angled at \( Q \). In each part calculate the length of \( QR \). Give each answer correct to 3 significant figures.

- a \( PQ = 7.3 \text{ cm}, \angle QPR = 68^\circ \)
- b \( PR = 17.2 \text{ m}, \angle QRP = 39^\circ \)
- c \( PR = 12.6 \text{ cm}, \angle QPR = 59^\circ \)

4 In triangle \( ABD \) the point \( C \) lies on \( AD \) so that \( BC \) and \( AD \) are perpendicular.

- a Using triangle \( ABC \), work out the length of
  i \( BC \)
  ii \( AC \)
  Give each answer correct to 3 significant figures.
- b Work out the length of \( CD \) correct to 3 significant figures.
- c Hence calculate the length of \( AD \) correct to 3 significant figures.
- d Calculate the area of triangle \( ABD \). Give your answer correct to the nearest cm\(^2\).

5 Calculate the length of \( BC \) in these triangles. Give each answer correct to 3 significant figures.

- a \( C \)
- b \( C \)
19.7 Finding angles using trigonometry

Example 10

Work out the size of each of the angles marked with letters.
Give each answer correct to 1 decimal place.

Solution 10

a

\[ \sin a = \frac{11.7}{15.9} = 0.7358... \]
\[ a = 47.379...^\circ \]
\[ a = 47.4^\circ \]

b

\[ \cos b = \frac{7.5}{16.1} = 0.4658... \]
\[ b = 62.235...^\circ \]
\[ b = 62.2^\circ \]

c

\[ \tan c = \frac{6.2}{9.7} = 0.6391... \]
\[ c = 32.585...^\circ \]
\[ c = 32.6^\circ \]
Exercise 19E

1 Use a calculator to find the value of $x$ in each of the following. Give each answer correct to 1 decimal place where necessary.

- a $\cos x^\circ = 0.5$
- b $\sin x^\circ = 0.43$
- c $\cos x^\circ = 0.6$
- d $\tan x^\circ = 0.96$
- e $\sin x^\circ = 0.8516$
- f $\tan x^\circ = 2.03$
- g $\sin x^\circ = 0.047$
- h $\tan x^\circ = \frac{7}{2}$

2 Work out the size of each of the marked angles. Give each answer correct to 1 decimal place.

3 Triangle $ABC$ is right-angled at $B$. Give each answer correct to 0.1°.

- a $AB = 8.9$ cm and $BC = 12.1$ cm. Calculate the size of angle $ACB$.
- b $BC = 15.5$ cm, $AC = 24.7$ cm. Calculate the size of angle $BAC$.
- c $AB = 6.3$ cm, $AC = 11.8$ cm. Calculate the size of angle $ACB$.

4 In triangle $ACD$ the point $B$ lies on $AD$ so that $CB$ and $AD$ are perpendicular.

- a Using triangle $ABC$ calculate the size of angle $ACB$. Give your answer correct to 1 decimal place.
- b Using triangle $BCD$ calculate the size of angle $BCD$. Give your answer correct to 1 decimal place.
- c Hence calculate the size of angle $ACD$. Give your answer to the nearest degree.

19.8 Trigonometry problems

Trigonometry can be used to solve problems. Sometimes Pythagoras’ theorem is needed as well. Some questions involve bearings (see Section 2.8).

Example 11

Two towns, Aytown and Beeville, are 40 km apart. The bearing of Beeville from Aytown is 067°.

- a Calculate how far north and how far east Beeville is from Aytown. Give your answers correct to 3 significant figures.
- b Calculate the distance between Aytown and Ceeham. Give your answer to the nearest km.
- c Calculate the bearing of Ceeham from Aytown. Give your answer to the nearest degree.
Solution 11

a

\[ e = 40 \sin 67° = 36.82... \]

Distance east = 36.8 km

\[ n = 40 \cos 67° = 15.629... \]

Distance north = 15.6 km

b

\[ AC^2 = AD^2 + DC^2 = 15.6^2 + 96.8^2 \]

\[ AC^2 = 9613.6 \]

\[ AC = 98.04... \]

Distance between Aytown and Ceeham is 98 km

\[ \tan DAC = \frac{96.8}{15.6} = 6.205... \]

\[ DAC = 80.8...° \]

\[ DAC = 81° \]

Bearing of Ceeham from Aytown is 081°
Exercise 19F
Where necessary give lengths correct to 3 significant figures and angles correct to 1 decimal place.

1. a Calculate the length of the line marked \(x\) cm.
   b Work out the size of the angle marked \(y^\circ\).

2. A ladder is 5 m long. The ladder rests against a vertical wall, with the foot of the ladder resting on horizontal ground. The ladder reaches up the wall a distance of 4.8 m.
   a Work out how far the foot of the ladder is from the bottom of the wall.
   b Work out the angle that the ladder makes with the ground.

3. The diagram shows the plans for the sails of a boat. Work out the length of the side marked
   a \(a\)
   b \(b\)
   c \(c\)

4. The diagram shows a vertical building standing on horizontal ground. The points \(A\), \(B\) and \(C\) are in a straight line on the ground. The point \(T\) is at the top of the building so that \(TC\) is vertical. The angle of elevation of \(T\) from \(A\) is 40° as shown in the diagram.
   a Work out the height, \(TC\), of the building.
   b Work out the size of the angle of elevation of \(T\) from \(B\).

5. The points \(P\) and \(Q\) are marked on a horizontal field. The distance from \(P\) to \(Q\) is 100 m. The bearing of \(Q\) from \(P\) is 062°. Work out how far
   a \(Q\) is north of \(P\)
   b \(Q\) is east of \(P\).

6. The diagram shows a circle centre \(O\). The line \(ABC\) is the tangent to the circle at \(B\).
   a Work out the radius of the circle.
   b Work out the size of angle \(OCB\).
7  A, B and C are three buoys marking the course of a yacht race.
   a  Calculate how far B is
      i  north of A       ii  east of A.
   b  Calculate how far C is
      i  north of B       ii  east of B.
   c  Hence calculate how far C is
      i  north of A       ii  east of A.
   d  Calculate the distance and bearing of C from A.

8  A rock, R, is 40 km from a harbour, H, on a bearing of 040°. A port, P, is 30 km from R on a bearing of 130°.
   a  Draw a sketch showing the points H, R and P and work out the size of angle HRP.
   b  Work out the distance HP.
   c  Work out the bearing of P from H.

9  The diagram shows an isosceles triangle. Calculate the area of the triangle. Give your answer to the nearest cm².

10 The diagram shows an isosceles trapezium.
   a  Work out the distance, h cm, between the two parallel sides of the trapezium.
   b  Work out the length of the longer parallel side of the trapezium.
   c  Calculate the area of the trapezium. Give your answer to the nearest cm².

Chapter summary

You should now know:

★ that in a right-angled triangle the side opposite the right angle is called the hypotenuse.
   It is the longest side in the triangle

★ Pythagoras’ theorem for right-angled triangles

\[ c^2 = a^2 + b^2 \]

★ trigonometric ratios for right-angled triangles

\[ \sin x^\circ = \frac{\text{opp}}{\text{hyp}} \]
\[ \cos x^\circ = \frac{\text{adj}}{\text{hyp}} \]
\[ \tan x^\circ = \frac{\text{opp}}{\text{adj}} \]

\[ \text{opp} = \text{hyp} \times \sin x^\circ \]
\[ \text{adj} = \text{hyp} \times \cos x^\circ \]
\[ \text{opp} = \text{adj} \times \tan x^\circ \]
You should now be able to:

- use Pythagoras’ theorem in right-angled triangles
  - to find the length of the hypotenuse when the lengths of the other two sides are known
  - to find the length of one of the shorter sides of the triangle when the lengths of the other two sides are known
- use Pythagoras’ theorem to find the length of a line segment, given the coordinates of the end points of the segment
- find the coordinates of the midpoint of a line segment, given the coordinates of the end points of the segment
- use trigonometry in right-angled triangles to find the length of an unknown side and to find the size of an unknown angle
- apply Pythagoras’ theorem and trigonometry to right-angled triangle problems, including bearings.

Chapter 19 review questions

1. \(ABC\) is a right-angled triangle. 
   \(AB = 8\) cm, \(BC = 11\) cm
   Calculate the length of \(AC\).
   Give your answer correct to 3 significant figures.

2. Angle \(MLN = 90^\circ\)
   \(LM = 3.7\) m
   \(MN = 6.3\) m
   Work out the length of \(LN\).
   Give your answer correct to 3 significant figures.

3. Work out the length in centimetres of \(AM\).
   Give your answer correct to 2 decimal places.

4. Ballymena is due west of Larne.
   Woodburn is 15 km due south of Larne.
   Ballymena is 32 km from Woodburn.
   a. Calculate the distance of Larne from Ballymena.
      Give your answer in kilometres, correct to 1 decimal place.
   b. Calculate the bearing of Ballymena from Woodburn.
5 Angle $ABC = 90^\circ$
Angle $ACB = 24^\circ$
$AC = 6.2\text{ cm}$
Calculate the length of $BC$.
Give your answer correct to 3 significant figures.

6 The diagram shows a rectangle drawn inside a circle.
The centre of the circle is at $O$.
The rectangle is 15 cm long and 9 cm wide.
Calculate the circumference of the circle.
Give your answer correct to 3 significant figures.

7 The diagram shows triangle $ABC$.
$BC = 8.5\text{ cm}$
Angle $ABC = 90^\circ$
Angle $ACB = 38^\circ$
Work out the length of $AB$.
Give your answer correct to 3 significant figures.

8 $ABD$ and $DBC$ are two right-angled triangles.
$AB = 9\text{ m}$
Angle $ABD = 35^\circ$
Angle $DBC = 50^\circ$
Calculate the length of $DC$.
Give your answer correct to 3 significant figures.

9 The diagram shows the positions of three telephone masts $A$, $B$ and $C$.
Mast $C$ is 5 kilometres due east of Mast $B$.
Mast $A$ is due north of Mast $B$ and 8 kilometres from Mast $C$.

a Calculate the distance of $A$ from $B$.
Give your answer in kilometres, correct to 3 significant figures.

b i Calculate the size of the angle marked $x^\circ$.
Give your angle correct to 1 decimal place.
ii Calculate the bearing of $A$ from $C$.
Give your answer correct to 1 decimal place.
iii Calculate the bearing of $C$ from $A$.
Give your bearing correct to 1 decimal place.
10 A and B are points on a centimetre grid.
A is the point (3, 2)
B is the point (7, 8)
a Calculate the distance AB.
Give your answer correct to 3 significant figures.
b Find the coordinates of the midpoint of AB.

11 ABCD is a trapezium.
AD is parallel to BC.
\[ \text{Angle } A = \text{angle } B = 90^\circ \]
\[ \text{AD} = 2.1 \text{ m} \]
\[ AB = 1.9 \text{ m} \]
\[ CD = 3.2 \text{ m} \]
Work out the length of BC.
Give your answer correct to 3 significant figures.

12 ABC is a right-angled triangle.
D is the point on AB such that AD = 3DB.
\[ AC = 2DB \text{ and angle } A = 90^\circ \]
Show that \( \sin C = \frac{k}{\sqrt{20}} \) where k is an integer.
Write down the value of k.

13 Ambletown, Bowtown and Comptown are three towns.
Ambletown is 9.6 km due west of Bowtown.
Bowtown is 7.4 km due south of Comptown.
Calculate the bearing of Ambletown from Comptown.
Give your answer correct to 1 decimal place.

14 ABCD is a quadrilateral.
\[ \text{Angle } BDA = 90^\circ, \quad \text{angle } BCD = 90^\circ, \quad \text{angle } BAD = 40^\circ \]
\[ BC = 6 \text{ cm}, \quad BD = 8 \text{ cm} \]
a Calculate the length of DC. Give your answer correct to 3 significant figures.
b Calculate the size of angle DBC. Give your answer correct to 3 significant figures.
c Calculate the length of AB. Give your answer correct to 3 significant figures.