This chapter is about two-dimensional shapes. Two-dimensional shapes are flat. Two-dimensional is often written as 2-D.

9.1 Drawing shapes
Here are some examples of making accurate drawings of shapes using a ruler, compasses and in some cases a protractor.

Example 1
Here is a sketch of a triangle.
Make an accurate drawing of the triangle.

Solution 1

An accurate drawing made with a ruler and compasses but not a protractor is called a construction. So Example 1 is a construction.

Step 1
Draw a line 10 cm long using a ruler. This is the base, $AB$, of the triangle. Sometimes in exam questions this base line will be drawn on the question paper.

Step 2
Using a ruler set your compasses to 8 cm. Put the point of the compasses on $A$ and draw an arc of a circle.

Step 3
Using a ruler set your compasses to 11 cm. Put the point of the compasses on $B$ and draw an arc of a circle.

Step 4
The point where the two arcs cross is the third vertex (corner), $C$, of the triangle. $CB$ is 11 cm long and $CA$ is 8 cm long so join $C$ to $A$ and $C$ to $B$ to complete triangle $ABC$. It is important that all construction lines can be seen. They should not be rubbed out.
Exercise 9A

1 Here is a sketch of triangle $ABC$.
   Use ruler and compasses to construct the triangle when
   a $AB = 10$ cm, $CA = 8$ cm and $CB = 9$ cm
   b $AB = 8.7$ cm, $CA = 9.4$ cm and $CB = 8.1$ cm
   c $AB = 4.6$ cm, $CA = 10.4$ cm and $CB = 7.9$ cm
   d $AB = 3.5$ cm, $CA = 12$ cm and $CB = 12.5$ cm.
   In each case measure the size of the largest angle of the triangle.

2 Use ruler and compasses to construct an equilateral triangle with sides of length 8 cm.

3 Here is a sketch of triangle $LMN$.
   a Make an accurate drawing of triangle $LMN$.
   b Measure the length of the side $LN$.
   Give your answer to the nearest 0.1 cm.

4 Here is a sketch of a shape (rhombus).
   Make an accurate drawing of the rhombus.

5 Here is a sketch of a shape (kite) $ABCD$.
   $AC = 10.8$ cm, $AB = AD = 8.2$ cm, $BC = DC = 4.7$ cm.
   Use ruler and compasses to construct the kite.

6 Here are sketches of three triangles.
   Make an accurate drawing of each triangle.

7 Triangle $ABC$ is isosceles. The sides $AB$ and $AC$ are both 5.6 cm long. The angles at $B$ and $C$ are both $38^\circ$.
   a Draw a sketch of triangle $ABC$ showing the lengths of sides $AB$ and $AC$ and the size of the angles at $B$ and $C$ on the sketch.
   b Make an accurate drawing of triangle $ABC$.
   c Measure the length of $BC$. Give your answer to the nearest 0.1 cm.
9.2 Special quadrilaterals

A quadrilateral has 4 sides. Some quadrilaterals have special names. The table shows some of the properties of special quadrilaterals.

<table>
<thead>
<tr>
<th>Quadrilateral</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>All sides equal in length</td>
</tr>
<tr>
<td></td>
<td>All angles are 90°</td>
</tr>
<tr>
<td>Rectangle</td>
<td>Opposite sides equal in length</td>
</tr>
<tr>
<td></td>
<td>All angles are 90°</td>
</tr>
<tr>
<td>Rhombus</td>
<td>All sides equal in length</td>
</tr>
<tr>
<td></td>
<td>Opposite sides parallel</td>
</tr>
<tr>
<td></td>
<td>Opposite angles equal</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>Opposite sides equal in length and parallel</td>
</tr>
<tr>
<td></td>
<td>Opposite angles equal</td>
</tr>
<tr>
<td>Trapezium</td>
<td>One pair of parallel sides</td>
</tr>
<tr>
<td>Isosceles trapezium</td>
<td>One pair of parallel sides</td>
</tr>
<tr>
<td></td>
<td>Non-parallel sides equal in length</td>
</tr>
<tr>
<td>Kite</td>
<td>Two pairs of adjacent sides equal in length</td>
</tr>
<tr>
<td></td>
<td>(adjacent means 'next to')</td>
</tr>
</tbody>
</table>

9.3 Perimeter and area of rectangles

The perimeter of a two-dimensional shape is the total distance around the edge or boundary of the shape.

As a perimeter is a distance the units of perimeter are the units of length. So a perimeter can be measured in millimetres (mm), centimetres (cm), metres (m) or kilometres (km) for example.

So the perimeter of a rectangle which is 4 cm long and 3 cm wide is 

\[(4 + 3 + 4 + 3) \text{ cm} = 14 \text{ cm}\].
A formula for the perimeter, \( P \), of a rectangle with length, \( l \), and width, \( w \), is
\[
P = l + w + l + w
\]
which simplifies to
\[
P = 2l + 2w \quad \text{or} \quad P = 2(l + w)
\]
The area of a two-dimensional shape is a measure of the amount of space inside the shape.

The area of a centimetre square is 1 square centimetre. This is written as \( 1 \text{ cm}^2 \)
The area of a square with sides of length 1 m (a metre square) is 1 square metre or \( 1 \text{ m}^2 \)

The diagram shows a rectangle. The length of the rectangle is 4 cm and its width is 3 cm.
The rectangle is divided up into centimetre squares.
There are 12 centimetre squares inside the rectangle so that the area of the rectangle is 12 cm\(^2\).
The number of squares inside the rectangle is \( 4 \times 3 = 12 \)
So to find the area of a rectangle multiply its length by its width.

**Area of a rectangle = length \times width**

The area, \( A \), of a rectangle with length, \( l \), and width, \( w \), is given by the formula
\[
A = lw
\]
For a square the width is equal to the length and so to find the area of a square, multiply the length of the side of the square by itself, that is, square it.

**Area of a square = length \times length**

The area, \( A \), of a square of side, \( l \), is given by the formula
\[
A = l \times l \quad \text{or} \quad A = l^2
\]

**Example 2**

A football pitch is a rectangle with a length of 120 m and a width of 75 m.
Work out  
\( a \) its perimeter  
\( b \) its area.

**Solution 2**

\( a \) Perimeter = \( 2 \times 120 + 2 \times 75 \)
\[
= 240 + 150
\]
\[
= 390 \text{ m}
\]
As the lengths are in metres the units of the perimeter are m.

\( b \) Area = \( 120 \times 75 \)
\[
= 9000 \text{ m}^2
\]
As the lengths are in metres the units of the area are m\(^2\).
9.4 Area of a parallelogram

To find the area of this parallelogram, remove the orange triangle and replace it at the other end of the parallelogram to make a rectangle.

The area of the parallelogram is equal to the area of a rectangle with the same base and the same height as the parallelogram.

The area, \( A \), of a parallelogram with base \( b \) and height \( h \) is given by the formula:

\[
A = bh
\]

**Example 3**

Work out the area of the parallelogram.

**Solution 3**

Area = \( 8.2 \times 5.8 \)  
Area = 47.56 mm\(^2\)

As the lengths are in millimetres the units of the area are mm\(^2\).

9.5 Area of a triangle

Start with this triangle. Join a congruent triangle to it as shown to make a parallelogram.

The area of the triangle is half the area of the parallelogram.

The area, \( A \), of a triangle with base \( b \), and height \( h \), is given by the formula:

\[
A = \frac{1}{2}bh
\]
Example 4

Work out the area of the triangle.

Solution 4

Area of a triangle = \( \frac{1}{2} \times \text{base} \times \text{height} \).

Area = \( \frac{1}{2} \times 6 \times 5 \)

Area = 15 cm²

9.6 Area of a trapezium

Start with this trapezium. Join a congruent trapezium to it as shown to make a parallelogram.

The area of the trapezium is half the area of the parallelogram.

The base of the parallelogram is the sum of the parallel sides of the trapezium. \( h \) is the perpendicular distance between the parallel sides.

Area of a trapezium = \( \frac{1}{2} \times \text{sum of parallel sides} \times \text{distance between them} \)

The area, \( A \), of a trapezium with parallel sides of length \( a \) and \( b \) and a distance, \( h \), between the parallel sides is given by the formula

\[
A = \frac{1}{2} (a + b)h
\]

Example 5

Work out the area of the trapezium.

Solution 5

Area = \( \frac{1}{2} \times (11 + 5) \times 7 \)

Area = \( \frac{1}{2} \times 16 \times 7 \)

Area = 56 cm²

Exercise 9B

1. Write down the names of the quadrilaterals with
   a. all sides the same length
   b. all angles equal
   c. both pairs of opposite sides parallel
   d. opposite angles equal

2. The length of a rectangle is 9 cm and its width is 4 cm. Work out a. the perimeter and b. the area.
3 The length of each side of a square is 7 cm.  
Work out a the perimeter and b the area.

4 Work out the areas of these triangles and parallelograms.

- **a**
  - Triangle
  - Base: 8 cm
  - Height: 10 cm

- **b**
  - Triangle
  - Base: 9 cm
  - Height: 9 m

- **c**
  - Parallelogram
  - Base: 7 cm
  - Height: 5 cm

- **d**
  - Parallelogram
  - Base: 6 mm
  - Height: 9 mm

- **e**
  - Triangle
  - Base: 12 cm
  - Height: 5 cm

5 Copy and complete this table.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Length</th>
<th>Width</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>7 cm</td>
<td>9 cm</td>
<td>63 cm²</td>
</tr>
<tr>
<td>Rectangle</td>
<td>10 cm</td>
<td></td>
<td>40 cm²</td>
</tr>
<tr>
<td>Rectangle</td>
<td>5 cm</td>
<td></td>
<td>30 cm²</td>
</tr>
</tbody>
</table>

6 Copy and complete this table.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Base</th>
<th>Height</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>5 cm</td>
<td>10 cm</td>
<td>25 cm²</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>8 cm</td>
<td>4 cm</td>
<td>32 cm²</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>7 cm</td>
<td></td>
<td>56 cm²</td>
</tr>
<tr>
<td>Triangle</td>
<td></td>
<td>8 cm</td>
<td>24 cm²</td>
</tr>
</tbody>
</table>

7 Work out the areas of these trapezia.

- **a**
  - Trapezium
  - Top base: 4 cm
  - Bottom base: 10 cm
  - Height: 5 cm

- **b**
  - Trapezium
  - Top base: 5 cm
  - Bottom base: 8 cm
  - Height: 6 cm

- **c**
  - Trapezium
  - Top base: 4 cm
  - Bottom base: 7 cm
  - Height: 5 cm

8 Work out the areas of these rectangles. Give answers correct to the nearest whole number.

- **a**
  - Rectangle
  - Length: 4 cm
  - Width: 3 cm

- **b**
  - Rectangle
  - Length: 8 cm
  - Width: 2 cm

- **c**
  - Rectangle
  - Length: 7 cm
  - Width: 4 cm
9.7 Problems involving areas

Questions sometimes involve using the areas of rectangles, squares, triangles and parallelograms.

Example 6

Work out the area of the shape.

Solution 6

Area of rectangle A = \(9 \times 7\)
\[= 63 \text{ cm}^2\]

Area of triangle B = \(\frac{1}{2} \times 4 \times 3\)
\[= 6 \text{ cm}^2\]

Area of shape = 63 + 6
\[= 69 \text{ cm}^2\]
Example 7

A rectangular wall is 450 cm long and 300 cm high. The wall is to be tiled. The tiles are squares of side 50 cm. How many tiles are needed?

Solution 7

Method 1

Number of tiles needed for the length = \( \frac{450}{50} = 9 \)

Number of tiles needed for the height = \( \frac{300}{50} = 6 \)

Number of tiles needed = 9 \times 6

Number of tiles needed = 54

Method 2

Area of wall = \( 450 \times 300 \) = 135,000 cm\(^2\)

Area of a tile = \( 50 \times 50 \) = 2,500 cm\(^2\)

Number of tiles = \( \frac{135,000}{2,500} = 54 \)

Exercise 9C

1. Work out a the area and b the perimeter of this shape. All the corners are right angles.

2. Work out the areas of these shapes.
3 Work out the area of this shape.

4 The diagram shows the floor plan of a room.
   Work out the area of the floor.
   Give the units of your answer.

5 The diagram shows a rectangular piece of yellow paper with a corner removed.
   Work out the area of the yellow paper that is left.

6 The floor of a room is a 5 m by 3 m rectangle. The carpet used to cover the floor completely costs £8.95 a square metre. Work out the cost of the carpet used.

7 Karl wants to make a rectangular lawn in his garden. He wants the lawn to be 30 m by 10 m. Karl buys rectangular strips of turf 5 m long and 1 m wide. Work out how many strips of turf Karl needs to buy.

8 Work out the area of the shaded region in this diagram.

9 A wall is a 300 cm by 250 cm rectangle. Tiles, which are squares of side 50 cm, are used to tile the wall. Work out how many tiles are needed.

10 Trevor is going to paint some doors in his house. Each door is a 2 m by 0.85 m rectangle and he is going to paint both sides of each door. Each tin of paint that he is going to use covers 8 m². Trevor wants to paint 20 doors. How many tins of paint does he need to buy?

11 A rectangle is 9 cm by 4 cm. A square has the same area as the rectangle. Work out the length of each side of the square.
9.8 Circumference of a circle

The circumference is the special name of the perimeter of a circle, that is, the distance all around it. Measure the circumference and diameter of some circular objects.

For each one, work out the value of\[\frac{\text{circumference}}{\text{diameter}}.\]

The answer is always just over 3

The value of\[\frac{\text{circumference}}{\text{diameter}}\] is the same for every circle, 3.142 correct to 3 decimal places.

The value cannot be found exactly and the Greek letter \(\pi\) is used to represent it.

So for all circles\[\frac{\text{circumference}}{\text{diameter}} = \pi\]

and \[\text{circumference} = \pi \times \text{diameter}\]

Using \(C\) to stand for the circumference of a circle with diameter \(d\)

\[\frac{C}{d} = \pi\] and \[C = \pi d\]

To find the circumference of a circle, multiply its diameter by \(\pi\)

Example 8

Work out the circumference of a circle with a diameter of 6.8 cm. Give your answer correct to 3 significant figures.

Solution 8

\[\pi \times 6.8\]

\[= 21.3628 \ldots\]

Circumference = 21.4 cm

When the radius rather than the diameter is given in a question, one way of finding the circumference is to double the radius to obtain the diameter and then multiply the diameter by \(\pi\).

Alternatively use the fact that a circle’s diameter \(d\) is twice its radius \(r\), that is, \(d = 2r\).

Replace \(d\) by \(2r\) in the formula \[C = \pi d\] giving \[C = \pi \times 2r\] which can be written as \[C = 2\pi r\].

Example 9

The circumference of a circle is 29.4 cm. Work out its diameter. Give your answer correct to 3 significant figures.
Solution 9

Method 1

\[ 29.4 = \pi d \]

Substitute 29.4 for \( C \) in the formula \( C = \pi d \).

\[ d = 29.4 \div \pi \]

Divide both sides by \( \pi \).

\[ = 9.3583... \]

Divide 29.4 by \( \pi \) and write down at least 4 figures of the calculator display.

Diameter = 9.36 cm

Round the diameter to 3 significant figures. The units are cm.

The formula \( C = \pi d \) can be rearranged with \( d \) as the subject and used to find the diameter of a circle if its circumference is given.

Dividing both sides of \( C = \pi d \) by \( \pi \) gives \( d = \frac{C}{\pi} \)

To find the diameter of a circle, divide its circumference by \( \pi \)

Method 2

\[ 29.4 \div \pi \]

Divide the circumference by \( \pi \).

\[ d = 9.3583... \]

Write down at least 4 figures of the calculator display.

Diameter = 9.36 cm

Round the diameter to 3 significant figures. The units are cm.

Exercise 9D

If your calculator does not have a \( \pi \) button, take the value of \( \pi \) to be 3.142

Give answers correct to 3 significant figures.

1. Work out the circumferences of circles with these diameters.
   - a 4.2 cm
   - b 9.7 m
   - c 29 cm
   - d 12.7 cm
   - e 17 m

2. Work out the circumferences of circles with these radii.
   - a 3.9 cm
   - b 13 cm
   - c 6.3 m
   - d 29 m
   - e 19.4 cm

3. Work out the diameters of circles with these circumferences.
   - a 17 cm
   - b 25 m
   - c 23.8 cm
   - d 32.1 cm
   - e 76.3 m

4. The circumference of a circle is 28.7 cm. Work out its radius.

5. The diameter of the London Eye is 135 m. Work out its circumference.

6. The tree with the greatest circumference in the world is a Montezuma cypress tree in Mexico. Its circumference is 35.8 m. Work out its diameter.

7. Taking the equator as a circle of radius 6370 km, work out the length of the equator.

8. The circumference of a football is 70 cm. Work out its radius.

9. A semicircle has a diameter of 25 cm.
   Work out its perimeter.
   (Hint: the perimeter includes the diameter)
10 A semicircle has a radius of 19 m. Work out its perimeter.

11 The diagram shows a running track. The ends are semicircles of diameter 57.3 m and the straights are 110 m long. Work out the total perimeter of the track.

12 A reel of cotton has a radius of 1.3 cm. The cotton is wrapped round it 500 times. Work out the total length of cotton. Give your answer in metres.

13 The radius of a cylindrical tin of soup is 3.8 cm. Work out the length of the label. (Ignore the overlap.)

14 The diameter of a car wheel is 52 cm.
   a. Work out the circumference of the wheel.
   b. Work out the distance the car travels when the wheel makes 400 complete turns. Give your distance in metres.

15 The big wheel of a ‘penny farthing’ bicycle has a radius of 0.75 m. Work out the number of complete turns the big wheel makes when the bicycle travels 1 kilometre.

9.9 Area of a circle

The diagram shows a circle which has been split up into equal ‘slices’ called sectors.

The sectors can be rearranged to make this new shape.

Splitting the circle up into more and more sectors and rearranging them, the new shape becomes very nearly a rectangle. The length of the rectangle is half the circumference of the circle. The width of the rectangle is equal to the radius of the circle. The area of the rectangle is equal to the area of the circle.

\[
\text{Area of circle} = \frac{1}{2} \times \text{circumference} \times \text{radius} = \frac{1}{2} \times 2\pi r \times r
\]

or \[
A = \pi r^2
\]

To find the area of a circle multiply \( \pi \) by the square of the radius

or \[
\text{Area of a circle} = \pi \times \text{radius} \times \text{radius}
\]

If the diameter rather than the radius is given in a question, the first step is to halve the diameter to get the radius.
The diameter of a circle is 9.6 m.  
Work out its area.  
Give your answer correct to 3 significant figures.

**Solution 10**

\[ 9.6 \div 2 = 4.8 \]  
Divide the diameter by 2 to get the radius.

\[ \pi \times 4.8^2 \]  
Square the radius and then multiply by \( \pi \).

\[ = 72.3822 \ldots \]  
Write down at least 4 figures of the calculator display.

\[ \text{Area} = 72.4 \, \text{m}^2 \]  
Round the area to 3 significant figures. The units are \( \text{m}^2 \)

**Exercise 9E**

If your calculator does not have a \( \pi \) button, take the value of \( \pi \) to be 3.142  
Give answers correct to 3 significant figures.

1. Work out the areas of circles with these radii.  
   \[ \begin{align*}  
   \text{a} & \quad 7.2 \, \text{cm} \\  
   \text{b} & \quad 14 \, \text{m} \\  
   \text{c} & \quad 1.5 \, \text{cm} \\  
   \text{d} & \quad 3.7 \, \text{m} \\  
   \text{e} & \quad 2.43 \, \text{cm} 
   \end{align*} \]

2. Work out the areas of circles with these diameters.  
   \[ \begin{align*}  
   \text{a} & \quad 3.8 \, \text{cm} \\  
   \text{b} & \quad 5.9 \, \text{cm} \\  
   \text{c} & \quad 18 \, \text{m} \\  
   \text{d} & \quad 0.47 \, \text{m} \\  
   \text{e} & \quad 7.42 \, \text{cm} 
   \end{align*} \]

3. The radius of a dartboard is 22.86 cm.  
   Work out its area.

4. The diameter of Avebury stone circle is 365 m.  
   Work out the area enclosed by the circle.

5. The radius of a semicircle is 2.7 m.  
   Work out its area.

6. The diameter of a semicircle is 8.2 cm.  
   Work out its area.

7. The diagram shows a running track.  
   The ends are semicircles of diameter 57.3 m and the straights are 110 m long.  
   Work out the area enclosed by the track.
8 The diagram shows a circle of diameter 6 cm inside a square of side 10 cm.
   a Work out the area of the square.
   b Work out the area of the circle.
   c By subtraction work out the area of the shaded part of the diagram.

9 The diagram shows a circle of radius 7 cm inside a circle of radius 9 cm.
Work out the area of the shaded part of the diagram.

10 The diagram shows a 8 cm by 6 cm rectangle inside a circle of diameter 10 cm.
Work out the area of the shaded part of the diagram.

9.10 Circumferences and areas in terms of $\pi$

Answers to questions involving the circumference or area of a circle are sometimes given in terms of $\pi$, which is exact, and not as a number, which is approximate.

Example 11
The diameter of a circle is 8 cm.
Find the circumference of the circle.
Give your answer as a multiple of $\pi$.

Solution 11
$\pi \times 8$
Circumference $= 8\pi$ cm

Example 12
The radius of a circle is 3 m.
Find the area of the circle.
Give your answer as a multiple of $\pi$.

Solution 12
$\pi \times 3^2 = \pi \times 9$
Area $= 9\pi$ m$^2$

Example 13
The diameter of a semicircle is 12 cm.
Find the perimeter of the semicircle.
Give your answer in terms of $\pi$.

Solution 13
The perimeter is the sum of the diameter and half the circumference.
$\frac{\pi \times 12}{2} = \frac{12\pi}{2} = 6\pi$
Perimeter $= 6\pi + 12$ cm

If the circumference of a circle is given as a multiple of $\pi$ its diameter can be found.
The circumference of a circle is $30\pi$ m. Find its radius.

**Solution 14**

$$d = \frac{30\pi}{\pi} = 30$$

To find the diameter divide the circumference by $\pi$.

$$\frac{30}{2} = 15$$

To find the radius, divide the diameter by 2.

Radius $= 15$ m

The units are m.

**Exercise 9F**

In questions 1–4, give the answers as multiples of $\pi$.

1. Find the circumference of a circle with a diameter of 7 m.
2. Find the area of a circle with a radius of 5 cm.
3. Find the circumference of a circle with a radius of 8 cm.
4. Find the area of a circle with a diameter of 20 m.
5. The diameter of a semicircle is 18 cm. Find the perimeter of the semicircle. Give your answer in terms of $\pi$.
6. The radius of a semicircle is 7 cm. Find the perimeter of the semicircle. Give your answer in terms of $\pi$.
7. The radius of a semicircle is 10 cm. Find its area. Give your answer as a multiple of $\pi$.
8. The circumference of a circle is $16\pi$ cm. Find its diameter.
9. The circumference of a circle is $30\pi$ m. Find its radius.
10. The circumference of a circle is $14\pi$ cm. Find its area. Give your answer as a multiple of $\pi$.

**9.11 Arc length and sector area**

An **arc** is part of the circumference of a circle.

A **sector** of a circle is formed by an arc and two radii. The perimeter of a sector is the sum of its arc length and two radii.

$90^\circ$ is $\frac{1}{4}$ of $360^\circ$ (a full turn) and so for a sector with an angle of $90^\circ$ at the centre the arc length is $\frac{1}{4}$ of the circumference of the whole circle.

The sector area is $\frac{1}{4}$ of the area of the whole circle.

For a sector with an angle of $70^\circ$ at the centre the arc length is $\frac{70}{360}$ of the circumference of the whole circle.

The sector area is $\frac{70}{360}$ of the area of the whole circle.

In general, for a sector with an angle of $x^\circ$ at the centre of a circle of radius $r$.

$$\text{arc length} = \frac{x}{360} \times 2\pi r$$

$$\text{sector area} = \frac{x}{360} \times \pi r^2$$
Example 15
Calculate the perimeter of this sector. Give your answer correct to 3 significant figures.

Solution 15
\[
\frac{80}{360} \times 2\pi \times 7 + 2 \times 7 = 9.773 \ldots + 14 = 23.773 \ldots
\]
Perimeter = 23.8 m

Example 16
Calculate the area of this sector. Give your answer correct to 3 significant figures.

Solution 16
\[
\frac{150}{360} \times \pi \times 5^2 = 32.72 \ldots
\]
Area = 32.7 cm²

9.12 Segment area
A segment of a circle is formed by a chord and an arc.

In the diagram, \(AB\) is a chord of a circle, centre \(O\).

Segment area = area of \(\text{sector } OAB\) — area of \(\text{triangle } OAB\).

Exercise 9G
Give answers correct to 3 significant figures unless the question states otherwise. If your calculator does not have a \(\pi\) button, take the value of \(\pi\) to be 3.142

1. Calculate the arc length of each of these sectors.

a. \(\frac{30}{360} \times 2\pi \times 4\) cm
b. \(\frac{70}{360} \times 2\pi \times 8\) m
c. \(\frac{130}{360} \times 2\pi \times 6\) cm
d. \(\frac{68}{360} \times 2\pi \times 10\) cm
2 Calculate the perimeter of each of these sectors.

3 Find the perimeter of each of these sectors. Give each answer in terms of \( \pi \).

4 Calculate the area of each of these sectors.

5 Find the area of each of these sectors. Give each answer as a multiple of \( \pi \).

6 a Find the area of the shaded segment.

b Find an expression for the area of the shaded segment when the radius of the circle is \( r \) cm. Give your answer in terms of \( r \) and \( \pi \).
9.13 Units of area

The diagram shows two congruent squares. The sides of square A are measured in metres and the sides of square B are measured in centimetres.

Square A is 1 m by 1 m so that the area of square A is \(1 \times 1 = 1 \text{ m}^2\).

As 100 cm = 1 m, square B is 100 cm by 100 cm so that the area of square B is \(100 \times 100 = 10000 \text{ cm}^2\).

The squares have the same area so \(1 \text{ m}^2 = 10000 \text{ cm}^2\).

There are also similar results for other units.

<table>
<thead>
<tr>
<th>Length</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cm = 10 mm</td>
<td>1 cm(^2) = 10 \times 10 = 100 mm(^2)</td>
</tr>
<tr>
<td>1 m = 100 cm</td>
<td>1 m(^2) = 100 \times 100 = 10000 cm(^2)</td>
</tr>
<tr>
<td>1 km = 1000 m</td>
<td>1 km(^2) = 1000 \times 1000 = 1 000 000 m(^2)</td>
</tr>
</tbody>
</table>

\[\text{divide by 100} \quad \text{multiply by 100} \quad \text{divide by 10 000} \quad \text{multiply by 10 000} \quad \text{divide by 1 000 000} \quad \text{multiply by 1 000 000} \]

**Example 17**

Change 4.6 m\(^2\) to cm\(^2\).

**Solution 17**

\[4.6 \text{ m}^2 = 4.6 \times 10000 \text{ cm}^2\]

\[= 46000 \text{ cm}^2\]

**Multiply the number of m\(^2\) by 10 000**

**Exercise 9H**

1. Change to cm\(^2\)
   - a 4 m\(^2\)
   - b 6.9 m\(^2\)
   - c 600 mm\(^2\)
   - d 47 mm\(^2\)

2. Change to m\(^2\)
   - a 5 km\(^2\)
   - b 0.3 km\(^2\)
   - c 40 000 cm\(^2\)
   - d 560 cm\(^2\)

3. a How many mm are there in 1 m?
   - b How many mm\(^2\) are there in 1 m\(^2\)?
   - c Change 8.3 m\(^2\) to mm\(^2\).

4. Find, in cm\(^2\), the area of a rectangle
   - a 3.2 m by 1.4 m
   - b 45 mm by 8 mm

5. Work out the area of this triangle in
   - a cm\(^2\)
   - b mm\(^2\)
You should now:

- be able to draw triangles and quadrilaterals accurately using ruler, protractor and compasses.

You should also know:

- the names and properties of special quadrilaterals
- the perimeter of a two-dimensional shape is the total distance around the edge or boundary of the shape
- the perimeter, $P$, of a rectangle with length, $l$, and width, $w$, is given by the formula
  \[ P = 2l + 2w \text{ or } P = 2(l + w) \]
- how to find the area of a rectangle using
  \[ A = lw \]
- how to find the area of a parallelogram using
  \[ A = bh \]
- how to find the area of a triangle using
  \[ A = \frac{1}{2}bh \]
- how to find the area of a trapezium using
  \[ A = \frac{1}{2}(a + b)h \]
- how to find the area and perimeter of a shape made from triangles and rectangles
- how to solve problems involving areas
- how to find the circumference of a circle using
  \[ C = \pi d \text{ and } C = 2\pi r \]
- how to find the diameter (or radius) of a circle if its circumference is known using
  \[ d = \frac{C}{\pi} \]
Chapter 9 review questions

1. Here is a sketch of a triangle. Use ruler and compasses to construct this triangle accurately. You must show all your construction lines.

2. The diagram shows a sketch of triangle \( \triangle ABC \).
   - \( BC = 7.3 \text{ cm} \)
   - \( AC = 8 \text{ cm} \)
   - Angle \( C = 38^\circ \)
   - a. Make an accurate drawing of triangle \( \triangle ABC \).
   - b. Measure the size of angle \( A \) on your diagram.

3. Work out the area of the triangle. Give the units with your answer.
4 This diagram shows the plan of a floor.
   a Work out the perimeter of the floor.
   b Work out the area of the floor.

5 Work out the area of the trapezium.

6 The diagram shows a shape.
   Work out the area of the shape.

7 The diagram shows a wall with a door in it.
   Work out the grey area.

8 Mary’s floor is a rectangle 8 m long and 5 m wide. She wants to cover the floor completely with carpet tiles.
   Each carpet tile is square with sides of length 50 cm. Each carpet tile costs £4.19
   Work out the cost of covering Mary’s floor completely with carpet tiles.

9 A table has a top in the shape of a circle with a radius of 45 centimetres.
   a Calculate the area of the circular table top.
      The base of the table is also in the shape of a circle.
      The circumference of this circle is 110 centimetres.
   b Calculate the diameter of the base of the table.

10 A rug is in the shape of a semicircle.
    The diameter of the semicircle is 1.5 m.
    Calculate the perimeter of the rug.
    Give your answer in cm correct to the nearest cm.

11 The diagram shows a shape.
    \( AB \) is an arc of a circle, centre \( O \).
    Angle \( AOB = 90^\circ \)
    \( OA = OB = 6 \) cm.
    Calculate the perimeter of the shape.
    Give your answer correct to 3 significant figures.

12 The diagram shows a circle of diameter 70 cm inside a square of side 70 cm.
    Work out the area of the shaded part of the diagram.
    Give your answer correct to 3 significant figures.
13 The diagram shows a right-angled triangle $ABC$ and a circle. $A$, $B$ and $C$ are points on the circumference of the circle. $AC$ is a diameter of the circle. The radius of the circle is 10 cm. $AB = 16$ cm and $BC = 12$ cm. Work out the area of the shaded part of the circle. Give your answer correct to the nearest cm$^2$. (1385 June 1999)

14 The diagram shows a shape made from a trapezium $ABCD$ and a semicircle with diameter $AB$. $AB = 18$ m $CD = 10$ m The total height of the shape is 21 m. Calculate the area of the whole shape. (Diagram NOT accurately drawn)

15 There is an infra-red sensor in a security system. The sensor can detect movement inside a sector of a circle. The radius of the circle is 15 m. The sector angle is $110^\circ$. Calculate the area of the sector. (1384 November 1994)

16 The diagram shows the sector of a circle, centre $O$. The radius of the circle is 9 cm. The angle at the centre of the circle is $40^\circ$. Find the perimeter of the sector. Leave your answer in terms of $\pi$. (Diagram NOT accurately drawn) (1387 June 2003)

17 The diagram shows a sector of a circle with a radius of $x$ cm and centre $O$. $PQ$ is an arc of the circle. Angle $POQ = 120^\circ$. Write down an expression in terms of $\pi$ and $x$ for 
   a the area of this sector 
   b the arc length of this sector. (Diagram NOT accurately drawn) (1387 November 2004)

18 a Change 7 m$^2$ to cm$^2$. b Change 50 000 mm$^2$ to cm$^2$. (1387 June 2005)